

Digital

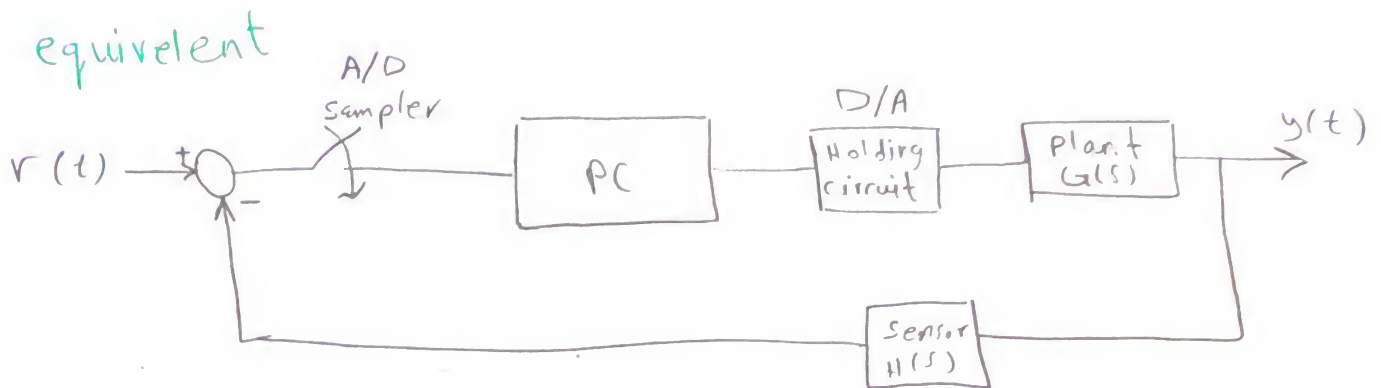
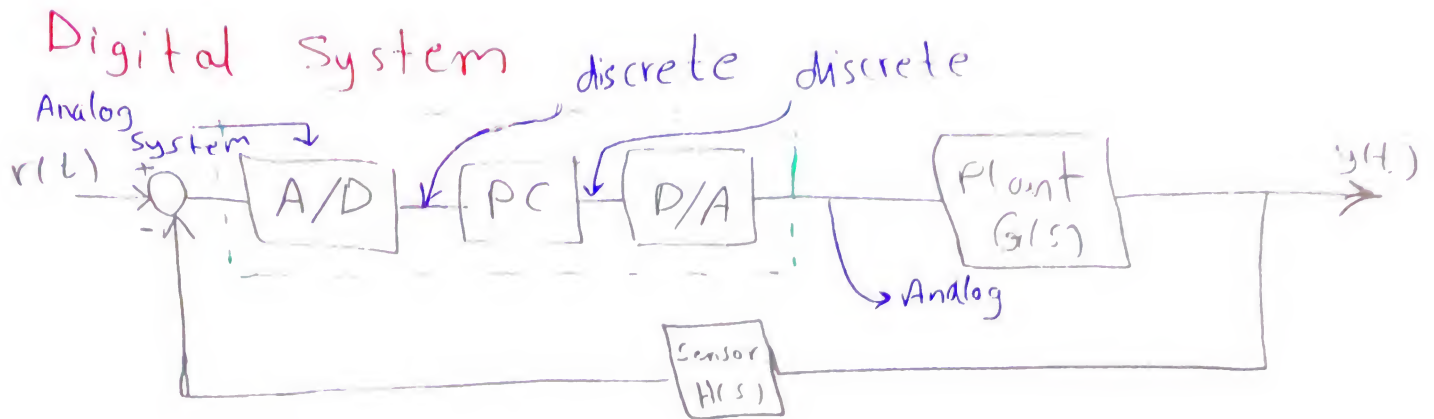
15/2/2016

الاسم

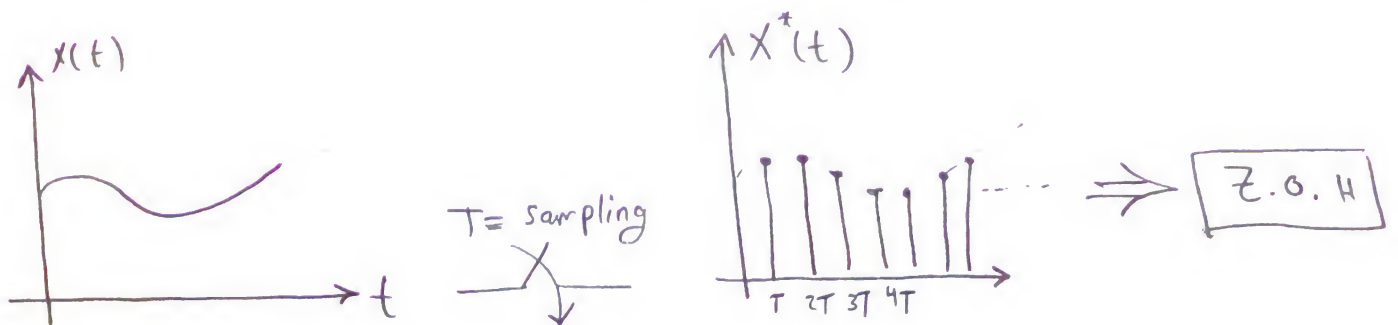
د. عرفة

حاضرة [1]

Revision on Z.T.

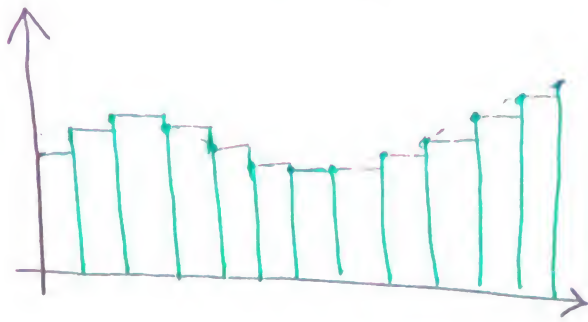


- * Digital System
- ① A/D
 - ② D/A converter \equiv Holding Circuit
 - Z.O.H (Zero order Hold)
 - F.O.H (First order Hold)
 - S.O.H (Second order Hold)



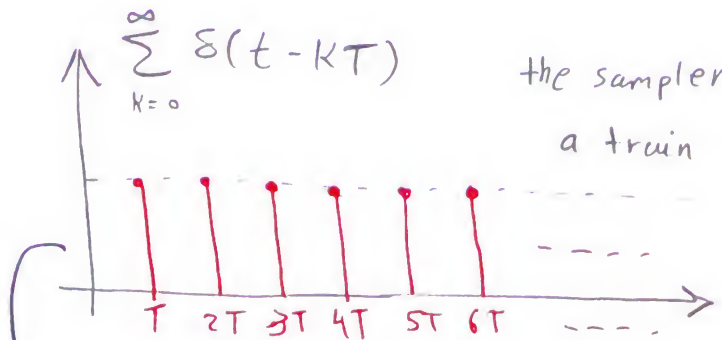
The output of the sampler is called star signal
 \equiv Sampled Signal

What Z.O.H does?



← Can be smoothed
Using L.P.F

* to get more accurate approximation of the input signal, we need more samples \Rightarrow requires higher frequency for sampling.

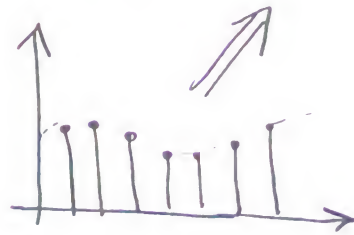


the sampler can be viewed as
a train of impulses

$$x^*(t) = \sum_{k=0}^{\infty} x(t) \delta(t - kT) = x(0) + x(T) + x(2T) + \dots$$

$K \rightarrow$ sampling no.

$T \rightarrow$ sampling period



$$x^*(t) = \sum_{k=0}^{\infty} x(kT) \delta(t - kT)$$

L.T.

$$X^*(s) = \sum_{k=0}^{\infty} x(kT) e^{-kTs}, \text{ let } z = e^{Ts}$$

$$X(z) = \left. X^*(s) \right|_{e^{Ts} = z} = \sum_{k=0}^{\infty} x(kT) z^{-k}$$

Ex (1)

$$x(t) = u(t) = 1$$

$$X(z) = \sum_{k=0}^{\infty} 1 \cdot z^{-k} = \sum_{k=0}^{\infty} z^{-k} = 1 + z^{-1} + z^{-2} + \dots$$

$$= \frac{1}{1 - z^{-1}} = \frac{z}{z - 1}$$

Ex (2)

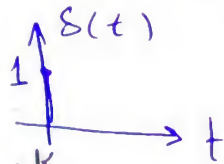
$$x(t) = e^{at}$$

$$X(z) = \sum_{k=0}^{\infty} e^{a k T} z^{-k} = 1 + e^{aT} z^{-1} + e^{2aT} z^{-2} + \dots$$

$$= \frac{1}{1 - e^{aT} z^{-1}} = \frac{z}{z - e^{aT}}$$

Ex (3)

$$x(t) = \delta(t)$$



$$X(z) = \sum_{k=0}^{\infty} \delta(kT) z^{-k} = \delta(0) + \cancel{\delta(0)} z^{-1} + \dots$$

$$= 1$$

Ex (4)

$$x(t) = a^t$$

$$X(z) = \sum_{k=0}^{\infty} a^{kT} z^{-k} = 1 + a^{kT} z^{-1} + a^{2kT} z^{-2} + \dots$$

$$= \frac{1}{1 - a^T z^{-1}} = \frac{z}{z - a^T}$$

Ex (5)

$$x(t) = t$$

$$X(z) = \sum_{k=0}^{\infty} kT z^{-k} = T z^{-1} + 2T z^{-2} + 3T z^{-3} + \dots$$

$$z X(z) = T + 2T z^{-1} + 3T z^{-2} + \dots$$

$$z X(z) - X(z) = T + T z^{-1} + T z^{-2} + T z^{-3} + \dots$$

⇒ continue

$$zX(z) - X(z) = \frac{T}{1 - z^{-1}}$$

$$(z-1)X(z) = \frac{T}{1 - z^{-1}} = \frac{Tz}{z-1}$$

$$X(z) = \frac{Tz}{(z-1)^2}$$

Ex (6)

$$x(t) = \sin \omega t$$

$$X(z) = \sum_{k=0}^{\infty} \sin(\omega kT) z^{-k}$$

$$= \sum_{k=0}^{\infty} \left[\frac{e^{j\omega kT} - e^{-j\omega kT}}{2j} \right] z^{-k}$$

$$= \frac{1}{2j} \left[\left(\sum_{k=0}^{\infty} e^{j\omega kT} z^{-k} \right) - \left(\sum_{k=0}^{\infty} e^{-j\omega kT} z^{-k} \right) \right]$$

$$= \frac{1}{2j} \left[\left(1 + e^{j\omega T} z^{-1} + e^{j2\omega T} z^{-2} + \dots \right) - \left(1 + e^{-j\omega T} z^{-1} + e^{-j2\omega T} z^{-2} + \dots \right) \right]$$

$$= \frac{1}{2j} \left[\frac{z}{z - e^{j\omega T}} - \frac{z}{z - e^{-j\omega T}} \right]$$

$$= \frac{1}{2j} \left[\frac{z(z - e^{-j\omega T}) - (z - e^{j\omega T})}{(z - e^{j\omega T})(z - e^{-j\omega T})} \right]$$

$$= \frac{z}{2j} \left[\frac{-e^{-j\omega T} + e^{j\omega T}}{z^2 - z(e^{j\omega T} + e^{-j\omega T}) + 1} \right]$$

$\xrightarrow{\text{sin}} \quad \quad \quad \xrightarrow{\text{cos}} \quad \quad \quad \xrightarrow{\text{cos}}$

$$= \frac{z \sin(\omega T)}{z^2 - 2z \cos(\omega T) + 1}$$

Ex (7)

$$X(t) = \cos \omega t$$

$$X(z) = \frac{z(z - \cos(\omega T))}{z^2 - 2z \cos(\omega T) + 1}$$

$$\boxed{1} \quad \mathcal{Z} \left[f_1(t) \Big|_{t=kT} \pm f_2(t) \Big|_{t=kT} \right] = F_1(z) \pm F_2(z)$$

$$\boxed{2} \quad \mathcal{Z} \left[e^{\pm aT} f(t) \Big|_{t=kT} \right] = F(z) \Big|_{z = z e^{\mp aT}}$$

$$\boxed{3} \quad \mathcal{Z} \left[a f(t) \Big|_{t=kT} \right] = a F(z)$$

$$\boxed{4} \quad \mathcal{Z} \left[a^t f(t) \Big|_{t=kT} \right] = F(z) \Big|_{z = \frac{z}{aT}}$$

$$\boxed{5} \quad \mathcal{Z} \left[t f(t) \Big|_{t=kT} \right] = -Tz \frac{dF(z)}{dz}$$

$\boxed{6}$ initial value : (the value at $t=0$)

$$f(0) = \lim_{t \rightarrow 0} f(t) = \lim_{z \rightarrow \infty} F(z)$$

Final value : (the value at $t = \infty$)

$$\begin{aligned} f(\infty) &= \lim_{t \rightarrow \infty} f(t) = \lim_{z \rightarrow 1} (z-1) F(z) \\ &= \lim_{z \rightarrow 1} (1-z^{-1}) F(z) \end{aligned}$$

$\frac{z-1}{z}$

$\boxed{7}$ shifting

$$\textcircled{1} \text{ delay : } \mathcal{Z} [x(k-n)] = z^{-n} X(z)$$

$$\textcircled{2} \text{ Advance : } \mathcal{Z} [x(k+n)] = z^n X(z) - z^n x(0) - z^{n-1} x(1) - \dots - z x(n-1)$$

ex:

$$z [x(k+1)] = z x(z) - z x(0)$$

$$z [x(k+2)] = z^2 x(z) - z^2 x(0) - z x(1)$$

$$z [x(k+3)] = z^3 x(z) - z^3 x(0) - z^2 x(1) - z x(2)$$

Proof:-

$$\begin{aligned} z [x(k+1)] &= \sum_{k=0}^{\infty} x(k+1) z^{-k} \\ &= x(1) + x(2) z^{-1} + x(3) z^{-2} + \dots \\ &= z \cdot z^{-1} [x(1) + x(2) z^{-1} + x(3) z^{-2} + \dots] \\ &= z [x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3} + \dots] \\ &= z \left[\underbrace{x(0) + x(1) z^{-1} + x(2) z^{-2} + \dots}_{x(z)} - x(0) \right] \\ &= z x(z) - x(0) \end{aligned}$$

* * You can prove that for other advances * *

Ex: $y(k+2) + 3y(k+1) + 2y(k) = \delta(k)$

z.t. \downarrow if $y(0)=0, y(1)=-1$, Solve for $y(k)$

$$z^2 Y(z) - z^2 y(0) - z y(1) + 3[z Y(z) - z y(0)] + 2 Y(z) = 1$$

$$z^2 Y(z) + z + 3z Y(z) + 2 Y(z) = 1$$

$$(z^2 + 3z + 2) Y(z) = 1 - z$$

$$\Rightarrow Y(z) = \frac{1-z}{z^2 + 3z + 2} = \frac{1-z}{(z+1)(z+2)}$$

$$= \frac{A_1}{(z+1)} + \frac{A_2}{(z+2)}$$

$$\left| \begin{array}{l} A_1 = \frac{2}{1} = 2 \\ A_2 = \frac{3}{-1} = -3 \end{array} \right.$$

$$Y(z) = \frac{2z z^{-1}}{z+1} - \frac{3z z^{-1}}{z+2}$$

$$I, Z.T = Z^{-1}.T, \Rightarrow y(k) = 2(-1)^{k-1} u(k-1) - 3(-2)^{k-1} u(k-1)$$

Ex: $F(z) = \frac{z(z+1)}{(z+2)(z+4)}$, find $Z^{-1}.T$.

$$F(z) = z \left[\frac{z+1}{(z+2)(z+4)} \right]$$

$$= z \left[\frac{A_1}{(z+2)} + \frac{A_2}{(z+4)} \right]$$

$$\left| \begin{array}{l} A_1 = \frac{-1}{2} \\ A_2 = \frac{-3}{-2} = \frac{3}{2} \end{array} \right.$$

$$F(z) = -0.5 \frac{z}{z+2} + \frac{1.5}{z+4} \Rightarrow Z^{-1}.T \quad **$$

$$f(k) = -0.5 (-2)^k + 1.5 (-4)^k$$

Report: Solve the example
for $T = 0.5$ sec

Hint: difference will be at **